Similarly the average length of lines drawn from P across the $\triangle PED$ is

$$a\log\left(\frac{\sqrt{[a^2+x^2]}+x}{a}\right)\div\tan^{-1}[x/a].$$

Hence the average length of lines drawn from P across the rectangle $PEDA$ is

$$\frac{2}{\pi}\left[x\log\left(\frac{\sqrt{[a^2+x^2]}+a}{x}\right)+a\log\left(\frac{\sqrt{[a^2+x^2]}+x}{a}\right)\right].$$

Hence the required average is

$$M=\frac{4}{a\pi}\int_0^a\left[x\log\left(\frac{\sqrt{[a^2+x^2]}+a}{x}\right)+a\log\left(\frac{\sqrt{[a^2+x^2]}+x}{a}\right)\right]dx$$

$$=\frac{6a}{\pi}\log[\sqrt{2}+1]-\frac{2}{\pi}\int_0^a\frac{xdx}{\sqrt{[a^2+x^2]}}=\frac{2a}{\pi}\{3\log\sqrt{[2+1]}-\sqrt{[2+1]}\}.$$

NOTE.—These solutions differ because both problems are stated in the indefinite form and the authors have assumed different laws of distribution. ED. F.

MISCELLANEOUS.

168. Proposed by W. J. GREENSTREET, M. A., Editor of The Mathematical Gazette, Stroud, England.

Sum to n terms, $\sin\alpha\sin\beta+\sin\alpha-\beta\sin\beta+\gamma+\sin\alpha-2\beta\sin\beta+2\gamma+\ldots$

Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.

The series intended is evidently

$$\sin\alpha\sin\beta+\sin[\alpha-\beta]\sin[\beta+\gamma]+\sin[\alpha-2\beta]\sin[\beta+2\gamma]+\ldots \text{ to } n \text{ terms.}$$

Since $2\sin\alpha\sin\beta=\cos[\alpha-\beta]-\cos[\alpha+\beta]$ we get, if S=the sum

$$S=\tfrac{1}{2}\{\cos[\alpha-\beta]+\cos[\alpha-2\beta-\gamma]+\cos[\alpha-3\beta-2\gamma]+\ldots \text{ to } n \text{ terms}\}$$
$$-\tfrac{1}{2}\{\cos[\alpha+\beta]+\cos[\alpha+\gamma]+\cos[\alpha-\beta+\gamma]+\ldots \text{ to } n \text{ terms}\}.$$

Let $\alpha+\beta=\theta$, $=[\beta+\gamma]=\phi$, $[\alpha+\beta]=\psi$, $-[\beta-\gamma]=\rho$.

$$\therefore S=\tfrac{1}{2}\{\cos\theta+\cos[\theta+\phi]+\cos[\theta+2\phi]+\ldots+ \text{ to } n \text{ terms}\}$$
$$-\tfrac{1}{2}\{\cos\psi+\cos[\psi+\rho]+\cos[\psi+2\rho]+\ldots+ \text{ to } n \text{ terms}\}.$$

$$\therefore\ S=\frac{\cos[\theta+\frac{n-1}{2}\phi]\sin\frac{n\phi}{2}}{2\sin\frac{\phi}{2}}-\frac{\cos[\psi+\frac{n-1}{2}\rho]\sin\frac{n\rho}{2}}{2\sin\frac{\rho}{2}}$$

$$=\frac{\cos\{\alpha-\beta-\frac{n-1}{2}[\beta+\gamma]\}\sin\frac{n[\beta+\gamma]}{2}}{2\sin\frac{\beta+\gamma}{2}}-\frac{\cos\{\alpha+\beta-\frac{n-1}{2}[\beta-\gamma]\}\sin\frac{n[\beta-\gamma]}{2}}{2\sin\frac{\beta-\gamma}{2}}.$$

Also solved by A. H. Holmes.

PROBLEMS FOR SOLUTION.

GEOMETRY.

319. Proposed by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.

Given the radii and the distances apart of the centers of three circles, to find the radii of the eight circles touching the three given circles.

MECHANICS.

204. Proposed by W. J. GREENSTREET, M. A., Editor of The Mathematical Gazette, Stroud, England.

A set of particles have coplanar motion due to mutual attractions. Each particle is now affected with a velocity V parallel to a fixed direction. How will this affect the angular momentum of the set about their centroid?

NUMBER THEORY AND DIOPHANTINE ANALYSIS.

147. Proposed by PROF. R. D. CARMICHAEL, Anniston, Ala.

If $4n+3$ is prime, $2(1, 2, 3, \ldots, 4n)+1\equiv0$ (mod. $4n+3$); and conversely. If $4n+3$ is prime, $(1, 2, 3, \ldots, 2n)^2-4\equiv0$ (mod. $4n+3$); and conversely.

AVERAGE AND PROBABILITY.

190. Proposed by PROF. R. D. CARMICHAEL, Anniston, Ala.

A line is drawn at random across a regular $2n$-gon; what is the chance that it crosses parallel sides?